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New Treatment of the Multiple Mutually Coupled Inductors to Improve the Modified Nodal Analysis in Harmonic Regime

I. INTRODUCTION

The modified nodal approach (MNA) is suitable to analyze a large class of analog networks for common and special technical applications, of the point of view of network complexity and working regime. We will consider the analog circuits having multiple mutually coupled inductors, like in electric machines or power systems modeling. Although one can use the modified nodal analysis for dynamic regimes of linear or nonlinear circuits with coupled inductors [1-4], it is already known that the analysis of harmonic steady-state regimes in presence of mutually coupled inductors requires some difficulties [3,6]. Two ways are available to perform the MNA in such situations: to obtain the matrix of circuit admittances, respectively to remove the mutual couplings using certain equivalent schemes with controlled sources. The first way, although is apparently simple, requires a matrix inversion that can be very unproductively for large scale circuits and multiple mutually coupled inductors; the second way is preferred. Therefore, we propose here an original algorithm based on the concept of *exclusive coupling* that substitutes the mutually coupled inductors through schemes with voltage controlled current sources. On the new equivalent circuit the modified nodal equations are built and solve without special difficulties. This modeling introduces some new circuit elements, but all of them are nodal analysis compatible (we note that a previously known modeling of mutually coupled inductors uses current controlled voltage sources that are not nodal analysis compatible elements; at the same time, such modeling comes with a lot of supplementary nodes in the cases of multiple couplings). It is an important advantage of our method of the point of view of the MNA. The method is thought from the perspective of a systematic algorithm that can be implemented in a dedicated computing program. In fact, we have created a computing program based on this technique. It performs the above modeling, then builds and solves the modified nodal equations. Finally, it performs the balance of active/reactive/apparent powers in order to prove the quality of the analysis. Some concrete applications are solved, one of them being described here; the results are validated through a SPICE simulation.

II. NODAL ANALYSIS COMPATIBLE MODEL OF A PAIR OF COUPLED INDUCTORS

Firstly, one considers a pair of magnetically coupled linear inductors (fig. 1).

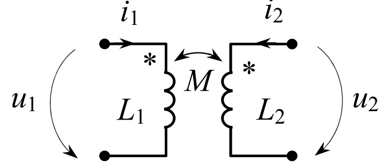

Fig. 1. Pair of magnetically coupled inductors

Starting of the known equations:

$$
\begin{aligned}
\underline{U}_1 &= j\omega L_1 \underline{I}_1 + j\omega M \underline{I}_2 \\
\underline{U}_2 &= j\omega M \underline{I}_1 + j\omega L_2 \underline{I}_2
\end{aligned} \tag{1}
$$

The currents $\underline{I}_1$ and $\underline{I}_2$ can be obtained by solving the equation system (1):

$$
\begin{aligned}
\underline{I}_1 &= \frac{L_2}{j\omega(L_1L_2 - M^2)}\underline{U}_1 - \frac{M}{j\omega(L_1L_2 - M^2)}\underline{U}_2 \\
\underline{I}_2 &= \frac{L_1}{j\omega(L_1L_2 - M^2)}\underline{U}_2 - \frac{M}{j\omega(L_1L_2 - M^2)}\underline{U}_1
\end{aligned} \tag{2}
$$

If the expressions (2) are put as follows:

$$
\begin{aligned}
\underline{I}_1 &= \underline{Y}_1 \underline{U}_1 - \underline{Y}_M \underline{U}_2 \\
\underline{I}_2 &= \underline{Y}_2 \underline{U}_2 - \underline{Y}_M \underline{U}_1
\end{aligned} \tag{3}
$$

where

$$
\underline{Y}_1 = \frac{1}{j\omega \dfrac{L_1L_2 - M^2}{L_2}}\ ;\ \ \underline{Y}_2 = \frac{1}{j\omega \dfrac{L_1L_2 - M^2}{L_1}} \tag{4}
$$

are the admittances of two uncoupled inductors and

$$
\underline{Y}_M = \frac{1}{j\omega \dfrac{L_1L_2 - M^2}{M}} \tag{5}
$$

is the transfer admittance of two voltage controlled current sources. Therefore, one can identify the corresponding modeling diagram shown in fig. 2, that contains a pair of uncoupled inductors and two voltage controlled current sources. All the elements are nodal analysis compatible.

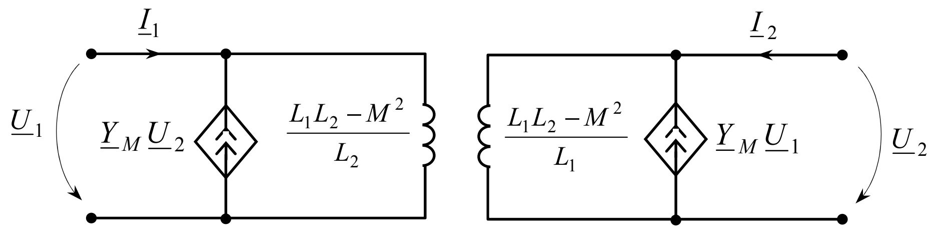

Fig. 2. Nodal analysis compatible model of single coupled inductors

III. MODELING OF THE MULTIPLE MUTUALLY COUPLED INDUCTORS

Let us consider the general case where each inductor is mutually coupled with all other inductors of the analyzed circuit. So, the inductor placed in the branch *k* is coupled with the inductors placed in the branches *j* ($j = \overline{1,\ p};\ j \neq k$). Therefore, the voltage of the inductor *k* is:

$$
\underline{U}_k = j\omega L_k \underline{I}_k + j\omega \sum_{\substack{j=1 \\ j \neq k}}^{p} M_{kj} \underline{I}_j \tag{5}
$$

Express in the same manner the voltages for all *p* coupled inductors and put them in the compact form as follows:

$$\underline{\boldsymbol{U}} = j\omega \boldsymbol{L}\,\underline{\boldsymbol{I}} \tag{6}$$

where we used the notations:

$$\begin{aligned} \underline{\boldsymbol{U}} &= \left[\underline{U}_1 \;\; \underline{U}_2 \ldots \underline{U}_k \ldots \underline{U}_p\right]^t \\ \underline{\boldsymbol{I}} &= \left[\underline{I}_1 \;\; \underline{I}_2 \ldots \underline{I}_k \ldots \underline{I}_p\right]^t \end{aligned} \tag{7}$$

$$\boldsymbol{L} = \begin{bmatrix} L_1 & \cdots & M_{1k} & \cdots & M_{1p} \\ \vdots & \ddots & \vdots & & \vdots \\ M_{k1} & \cdots & L_k & \cdots & M_{kp} \\ \vdots & & \vdots & \ddots & \vdots \\ M_{p1} & \cdots & M_{pk} & \cdots & L_p \end{bmatrix} \tag{8}$$

Obtain the currents simply by solving the system (6):

$$\underline{\boldsymbol{I}} = \underline{\boldsymbol{Y}} \cdot \underline{\boldsymbol{U}} \tag{9}$$

The *k*-th term of the vector $\underline{\boldsymbol{I}}$ is the current of the inductor *k*:

$$\underline{I}_k = \underline{Y}_k \underline{U}_k - \sum_{\substack{j=1 \\ j \neq k}}^{p} \underline{Y}_{kj} \underline{U}_j \tag{10}$$

The equation (10) permits us to identify the modeling diagram of the inductor *k* (shown in fig. 3).

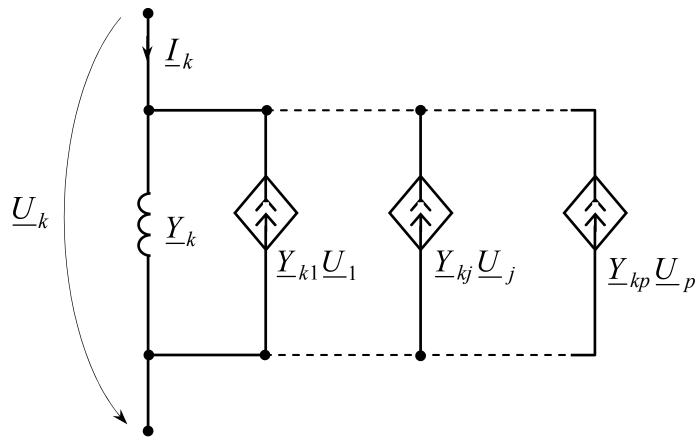

Fig. 3. Nodal analysis compatible model of a multiple coupled inductor

It contains an uncoupled inductor and $p-1$ voltage controlled current sources. The admittance $\underline{Y}_k$ and the transfer factors of the voltage controlled current sources $\underline{Y}_{kj}$ of the modeling diagram are the elements on the line *k* of the admittance matrix $\underline{\boldsymbol{Y}}$ of (9). All the elements of the model are nodal analysis compatible.

IV. BUILDING THE MODIFIED NODAL EQUATIONS

Starting from the circuit with uncoupled inductors obtained as above, the modified nodal equations can be put as:

$$\left[\begin{array}{c:c:c} \boldsymbol{A}_p\underline{\boldsymbol{Y}}_p\boldsymbol{A}_p^{\mathrm{t}} + \boldsymbol{A}_{Jc}\underline{\boldsymbol{Y}}_c\boldsymbol{A}_J^{\mathrm{t}} & \boldsymbol{A}_E + \boldsymbol{A}_{Jc}\boldsymbol{B}_c & \boldsymbol{A}_{Ec} \\ \hdashline \boldsymbol{A}_E^{\mathrm{t}} & \mathbf{0}_{l_E \times l_E} & \mathbf{0}_{l_E \times l_{Ec}} \\ \hdashline \boldsymbol{A}_{Ec}^{\mathrm{t}} + \boldsymbol{A}_c\boldsymbol{A}_J^{\mathrm{t}} & \underline{\boldsymbol{Z}}_c & \mathbf{0}_{l_{Ec} \times l_{Ec}} \end{array}\right] \cdot \begin{bmatrix} \underline{\boldsymbol{V}} \\ \underline{\boldsymbol{I}}_E \\ \underline{\boldsymbol{I}}_{Ec} \end{bmatrix} = -\begin{bmatrix} \boldsymbol{A}_J\underline{\boldsymbol{I}}_J \\ \underline{\boldsymbol{E}} \\ \mathbf{0}_{l_{Ec} \times 1} \end{bmatrix}, \qquad (11)$$

The equation system (11) can be built systematically by simple matrix operations, using the following matrices: $\boldsymbol{A}_p$, $\boldsymbol{A}_E$, $\boldsymbol{A}_J$, $\boldsymbol{A}_{Ec}$, $\boldsymbol{A}_{Jc}$, $\boldsymbol{A}_J$ are partitions of the branch-node incidence matrix according to the category of circuit elements; $\underline{\boldsymbol{Y}}_p$ is the matrix of admittances of passive branches; $\underline{\boldsymbol{Y}}_c$ is the matrix of transfer admittances of the voltage controlled current sources; $\underline{\boldsymbol{Z}}_c$ is the matrix of transfer impedances of the current controlled voltage sources; $\boldsymbol{A}_c$, $\boldsymbol{B}_c$ are the matrices of transfer factor of homogeneous controlled sources; $\underline{\boldsymbol{I}}_J$, $\underline{\boldsymbol{E}}$ are the vectors of the independent sources (circuit excitations); $\underline{\boldsymbol{V}}$, $\underline{\boldsymbol{I}}_E$, $\underline{\boldsymbol{I}}_{Ec}$ are the vectors of variables.

V. EXAMPLE

Let us consider the circuit below (fig. 4) working in harmonic regime; it will be analyzed using our dedicated computing program based on the described modeling of multiple mutually coupled inductors. The results will be verified through a SPICE simulation.

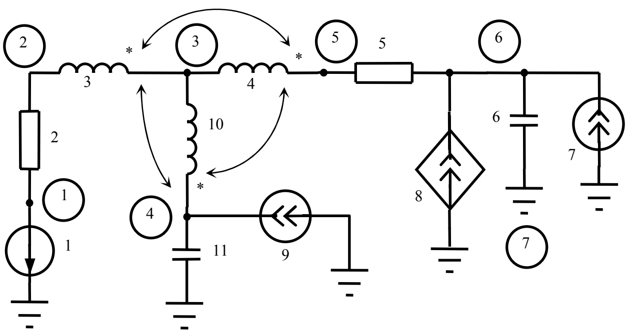

Fig. 4.Example of a circuit with multiple mutually coupled inductors

	A	B	C	D	E	F
1	E	1	1	7	100	0
2	R	2	1	2	10	
3	L	3	3	2	1.00E-01	
4	L	4	5	3	1.00E-01	
5	R	5	5	6	10	
6	C	6	6	7	1.00E-04	
7	J	7	7	6	10	0
8	JCU	8	7	6	0.8	9
9	J	9	7	4	0	0
10	L	10	4	3	0.025	
11	C	11	4	7	1.00E-04	
12	M	3	4	5.00E-03		
13	M	3	10	2.50E-03		
14	M	4	10	2.50E-03		
15	f	50				
16						

Fig. 5. Input data file

The entry data are given as a text file (or Excel table) shown in the figure 5. Each circuit element, as well as each mutually coupling, is described on its own line as follows:
a) for the bipolar elements of type: type (key word in column A), branch index (number in column B), initial node (number in column C), final node (number in column D), value of the parameter (number in column E); the phase is specified in column F for each independent harmonic source;
b) for controlled sources: type (key word in column A), controlled branch index (number in column B), initial node of the controlled branch (number in column C), final node

(number in column D), value of the transfer parameter (number in column E, controlling branch index (number in column F); the model of the controlling branch must be an independent current source if the controlling quantity is a voltage, respectively an independent voltage source if the controlling quantity is a current.
c) for the mutually coupling: the key word M (column A), the indexes of the coupled inductors (columns B and C), the mutual inductance expressed in Henry (column D).
The last line contains the signals frequency expressed in Hertz.
Our computing program has given the following results:

Branch currents:

I1 = 0.94418-3.0868i ==> 3.228 [A] / -72.9923 [deg]
I2 = -0.94418+3.0868i ==> 3.228 [A] / 107.0077 [deg]
I3 = 0.94418-3.0868i ==> 3.228 [A] / -72.9923 [deg]
I4 = 0.96292-2.7298i ==> 2.8946 [A] / -70.5698 [deg]
I5 = -0.96292+2.7298i ==> 2.8946 [A] / 109.4302 [deg]
I6 = -0.054443+3.2071i ==> 3.2075 [A] / 90.9726 [deg]
I7 = 10 ==> 10 [A] / 0 [deg]
I8 = -9.0915+0.4773i ==> 9.104 [A] / 176.9947 [deg]
I9 = 0 ==> 0 [A] / 0 [deg]
I10 = -0.018744-0.35702i ==> 0.35751 [A] / -93.0053 [deg]
I11 = 0.018744+0.35702i ==> 0.35751 [A] / 86.9947 [deg]

Branch voltages::

U1 = -100 ==> 100 [V] / 180 [deg]
U2 = -9.4418+30.868i ==> 32.2797 [V] / 107.0077 [deg]
U3 = 96.97459+29.66228i ==> 101.4097 [V] / 17.0077 [deg]
U4 = 86.0388+30.2364i ==> 91.1971 [V] / 19.3629 [deg]
U5 = -9.62923+27.2977i ==> 28.9463 [V] / 109.4302 [deg]
U6 = 102.0844+1.73297i ==> 102.0991 [V] / 0.97255 [deg]
U7 = -102.0844-1.73297i ==> 102.0991 [V] / -179.0274 [deg]
U8 = -102.0844-1.73297i ==> 102.0991 [V] / -179.0274 [deg]
U9 = -11.3644+0.596628i ==> 11.3801 [V] / 176.9947 [deg]
U10 = 4.948+0.60907i ==> 4.9854 [V] / 7.0174 [deg]
U11 = 11.3644-0.596628i ==> 11.3801 [V] / -3.0053 [deg]

Power balance

(powers given by the sources / powers absorbed by passive elements / error)
S = 1161.9344 VA / 1161.9344 VA / 0 %
P = 1115.262 W / 1115.262 W / 0 %
Q = 326.0094 VAr / 326.0094 VAr / 1.7436e-014 %

The obtained results are rigorously verified by the power balance.
In order to verify once again the results, a SPICE (commercial version ICAP4 from Intusoft) simulation is performed. The equivalent diagram built under SPICE is shown in figure 6; it contains a few of test points.

The SPICE input file is:

```
D:\Craiova\couplages
*SPICE_NET
.AC LIN 2 50 51
.PRINT AC VR(1) VI(1) IR(V4) II(V4)  IR(V3) II(V3)
*ALIAS  V(8)=U9
*ALIAS  I(V1)=ISURSA
.PRINT AC  I(V3) IP(V3) V(8)  VP(8)
.PRINT AC  I(V1) IP(V1) V(3)  VP(3)
L1 3 10 0.1
L3 8 3 0.025
C1 5 0 100U
I1 0 5 AC 10
```

K1 L1 L3 0.05
L2 4 3 0.1
K2 L2 L3 0.05
K3 L1 L2 0.05
G1 0 7 0 8 0.8
V3 7 5
R1 10 15 10
R2 4 5 10
I2 0 8 0
C2 8 0 100E-6
V1 0 15 AC 100
.END

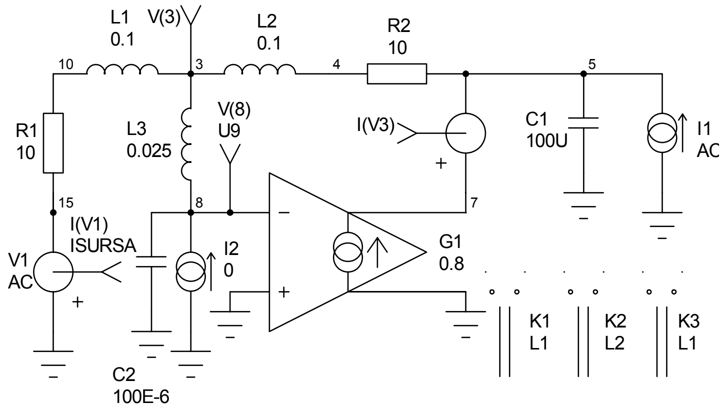

Fig. 6. SPICE diagram

We find some differences of 0.6 – 1 % between our results and the SPICE output. As an example, see the SPICE result obtained for the current of the branch 8, measured with the ammeter I(V3) (see fig. 6):

FREQUENCY	I(V3)	IP(V3)
5.000000e+001	9.183155e+000	1.767369e+002

Let compare it with the current I8 computed by our program (shown in bold in the result table above).

VI. CONCLUSION

The above described algorithm is generally applicable to any linear circuit excited by harmonic signals, having the same frequency. The complexity and the topological configuration of the analyzed circuit are not practically limited. The proposed modeling of the multiple mutually coupled inductors does not append any circuit element that can add supplementary variables in the modified nodal equations system (the equivalent circuit contains only nodal analysis compatible elements). Therefore, the method works with the smallest number of variables in comparison with other known methods (it is well known from the technical literature the modeling of the coupled inductors through current controlled voltage sources that introduce two supplementary variables in the MNA equation system).

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